Three-dimensional numerical modeling of hydrostatic tests of porous rocks in a triaxial cell

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It is known that there is stress concentration at specimen ends during experimental rock tests. This causes a deviation of the measured nominal (average) stresses and strains from the actual ones, but it is not completely clear how strong it could be. We investigate this issue by numerical modeling of the hydrostatic tests using a reasonably simple constitutive model that reproduces the principal features of the behavior of porous rocks at high confining pressure, $P_c$. The model setup includes the stiff (steel) platens and the cylindrical model rock specimen separated from the platens by the frictional interfaces with friction angle $\phi_{int}$. The whole model is subjected to the quasi-statically increasing normal stress $P_c$. During this process, the hydrostats $P_c(\varepsilon)$ are computed in the same way as in the real tests ($\varepsilon$ is the average volume strain). The numerical hydrostats are very similar to the real ones and are practically insensitive to $\phi_{int}$. On the contrary the stresses and strains within the specimen, are extremely sensitive to $\phi_{int}$. They are very heterogeneous and are characterized by a strong (proportional to $\phi_{int}$) along-axis gradient, which evolves with deformation. A strong deviation of the stress state at the specimen ends from the isotropic state results in inelastic deformation there at early loading stages. It follows that the nominal stresses and strains measured in the experimental tests can be very different from the actual ones, but they can be used to calibrate constitutive models via numerical simulations.

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1. Introduction

Poor knowledge of the constitutive properties of geomaterials (or more generally, granular, frictional, and dilatant/compactive materials) is the principal obstacle to modeling their deformation and failure necessary for various applications. There is a very extensive literature from different communities (physics, plasticity, structural engineering, rock and soil mechanics) on the constitutive modeling of geomaterials within a purely phenomenological framework [1–7], or micromechanical models based on various approaches, hypotheses, and assumptions [8–17], to mention only a few papers. Despite all that, there are still open questions which make it difficult to formulate adequate and sufficiently simple models. This reflects the very complex and not completely understood nature of the deformation of geomaterials, which suggests that models cannot be simple and highlights the importance of the experimental studies for further progress.

Apart from the fact that experimental data are limited and are mostly from a single loading configuration tests (axisymmetric compression), it is not completely clear how far the nominal strains and stresses measured (calculated) in the experimental tests correspond to the actual strains and stresses within the tested specimens. Both nominal and real stresses and strains are respectively equal only when the specimen is strained strictly uniformly, i.e., before the onset of strain localization and if the boundary effects at the specimen ends are negligible. Several experimental, theoretical, and numerical studies show that these effects can be very significant [18–24]. The principal practical conclusion from these studies is that the interface friction between the specimen and loading platens must be considerably reduced to obtain meaningful mechanical measurements. An efficient way to achieve such a reduction is to lubricate the specimen-platen contacts using adapted lubricants (both for the tested rock and the loading conditions), but even in this case the end effects can be significant. They grow dramatically with increasing axial stress $\sigma_{ax}$, indicative of the normal stress between the specimen and platens. For sandstone, for example, such an increase starts at $\sigma_{ax} = 60$ MPa [23,25].

To appreciate the impact of the end conditions on the stress-state within the specimen, Peng [18] and Brady [19,20] used purely elastic analysis, while Pellegrino et al. [23] and Albert and Rudnicki [24] carried out finite element modeling of the specimen compression with zero [23] and small confinement (corresponding to brittle deformation) [24]. In both cases, the numerical simulations were 2-D and used the Drucker–Prager constitutive model.
In the present study we investigate high-pressure deformation conditions by simulating numerically the hydrostatic tests in three dimensions. This work has been motivated by numerous recent studies of deformation of porous rocks under high pressure investigating the compaction mechanisms and formation of the compaction bands. The hydrostatic loading configuration has been chosen as the simplest one in the sense that it does not lead to the mechanical instabilities and the associated difficulties of the numerical analysis. The modeling setup was designed to be as close to the real experimental conditions as possible. Not only specimens of porous rocks are simulated but also the steel loading platens contacting with the specimen through frictional interfaces. An original constitutive model with a single cap-type yield function \( F(\sigma, \sigma_m, \gamma^p) \) is introduced that meets the requirement of being as simple as possible but capturing at the same time the principal features of the behavior of porous rocks at high confining pressure \( P_c \) (\( \tau \) is the von Mises stress, \( \gamma^p \) is the accumulated inelastic equivalent shear strain, and \( \sigma_m \) is the mean stress). This model has been implemented into the finite-difference 3-D time-matching explicit dynamic code Flac3D. The hydrostats (plots \( P_c(\varepsilon) \)) obtained in the numerical models reproduce very well the corresponding experimental curves (\( \varepsilon \) is the volume strain average over the specimen). Surprisingly, they are practically insensitive to the specimen–platens interface friction angle \( \phi_{int} \), while stresses and strains within the model are, on the contrary, strongly dependent on this parameter. The results obtained clearly show that the ‘global’ (external) measurements of \( \sigma_m \) (which is supposed to correspond to \( P_c \)) and \( \varepsilon \) provide only the first order estimation of the stresses and strains within the specimen and can be very different from the actual values. This is particularly true at the specimen ends where the compactive inelastic deformation is initiated first and then propagates toward the specimen center, forming a sort of expanding compaction zones. It follows that ideally the laboratory measurements should be coupled with the numerical simulations to define the stress-state of the specimen. The global measurements can serve to validate and calibrate the constitutive model used.

2. Mechanical data from rock tests

The hydrostatic experimental tests are conducted in the same way as the conventional triaxial tests. The difference is that the isotropic loading of a jacketed cylindrical specimen with the steel platens (Fig. 1a) in the pressure cell is not followed by the additional axial loading. The confining pressure \( P_c \) is usually increased to large values to cause grain crushing. Basically two principal parameters are measured: \( P_c \) (the pressure of a liquid in the pressure cell), and the average over the whole specimen (nominal) volume strain \( \varepsilon \). The latter is computed from the reduction of the specimen volume (equal to the water volume squeezed from the saturated specimen during the loading) or from the axial and radial displacements of points on the specimen surface measured with the LVDTs (or other) gauges.

The examples of the experimental hydrostats in Fig. 2 show that they all have the same trend (the same slope change with \( P_c \)). Firstly the slope increases due to the adjustments of the contact between the specimen and platens, but mainly due to microscale processes such as progressive crack closure and grain boundary compression, which corresponds to a nonlinear elasticity \([26]\). Then, the slope remains more or less constant (linear elasticity). After reaching a certain pressure \( P_c \approx P^* \), there is a rapid reduction of the slope. This pressure typically corresponds to the onset of grain crushing \([27]\), although in a granular rock analog material GRAM1 (inset in Fig. 2) there is no grain crushing as the pressure is too small. The rapid slope reduction at \( P_c \approx P^* \) is caused in this material by the breakage of grain bonds and grain reorganization resulting in the inelastic volume reduction (compaction). It follows that the rapid reduction of the hydrostat slope at \( P_c \approx P^* \) is due to the reduction of the dilatancy factor \( \beta \) (negative at high \( P_c \)), whatever the micromechanism of this reduction, grain breakage/pore collapse or grain reorganization or both. This independence of (or low sensitivity to) micromechanics is important for the constitutive modeling and is also confirmed by different microstructures of the compaction bands forming at high \( P_c \) and very

![Fig. 1.](image-url) (a) Set up of the hydrostatic tests both in the laboratory and the presented numerical models. (b) Numerical grid in the numerical models. 1. Cylindrical specimen; 2. Stiff (steel) platens; 3. Lubricant layers and or low-friction gaskets in the laboratory tests, and frictional interfaces in the numerical models. Radius of the model specimens is 2 cm and their height is 8 cm. The platen thickness in the models is 1.2 cm. Their Young modulus is 200 GPa and Poisson ratio 0.3.
negative $\beta$: in hard (well cemented) rocks the underlying mechanism is pore collapse and grain crushing [27–29], while in low cohesion materials, these bands form practically without grain crushing both in natural/geological conditions [31] and in a synthetic material GRAM1 deformed in the laboratory [30].

The low-slope linear segments of most of the hydrostats in Fig. 2 are followed by a new slope increase with $P_c$. In the cases where the slope increase is not observed, the relative volume reduction ($\epsilon$) is quite small (Fig. 2) and probably has not yet reached high enough values for the slope reproduction. It is obvious that a constitutive model for porous rocks should reproduce these observations from the hydrostatic tests.

3. Constitutive model

3.1. Yield function

The experimental data suggest a quadratic dependence of yield function $F$ on $\sigma_m$. In most cases an elliptic shape of the cap envelope is assumed [27], although other shapes with smaller negative slope were suggested as well [29,32]. It should be noted that it is very difficult reliably to define from the experimental data the limits of the elastic domain (or the initial yield envelop) in the stress space. The error margin can be significant since it is difficult to spot the onset of inelastic straining which is not sharp but very progressive [33]. Following the experimental data, the quadratic (either elliptic or parabolic) dependence on $\sigma_m$ is usually adopted in the constitutive models dealing with high-pressure deformation regimes [7,34–37].

The quadratic dependence is also assumed in this work

$$F = \tau - [Q_1(\gamma^p)\sigma_m^2 + Q_2(\gamma^p)\sigma_n + Q_3(\gamma^p)]\quad (1)$$

where $\tau$ is the von Mises stress, $\gamma^p$ is the equivalent inelastic shear strain, and $Q_i$ ($i = 1, 2, 3$) are the material parameters that are functions of $\gamma^p$. The main difficulty is the definition of these functions. Based on the numerous preliminary numerical simulations, we propose the following functional form:

$$Q_i(\gamma^p) = A_i + B_i(\gamma^p)^n, \quad (2)$$

where $A_i$, $B_i$, and $n$ are constant material parameters with $n = 0.5$.

Coefficients $A_i$ define the geometry of the initial (corresponding to $\gamma^p = 0$) yield function. This geometry was shown for porous sandstones to approximately scale with $P^*$ so that the coefficients $k_{mk} = \sigma^p_{mk}/P^*$ and $k_{sp} = \tau^p/\gamma^p$ are independent of $P^*$ to a first approximation [27]. Here $\tau^p$ and $\sigma^p_{mk}$ are the coordinates of the peak of the initial yield envelope/function. Considering that at the peaks $\partial F/\partial \sigma_m = 0$ and that at $\sigma_m = P^*/\tau = 0$, one can easily obtain expressions for $A_i$:

$$A_1 = \frac{-k_{sp}}{P^*(k_{sp} - 1)^2}, \quad A_2 = -\frac{2k_{mk}k_{sp}}{(k_{sp} - 1)^2}, \quad A_3 = -\frac{k_{mk}P^*(2k_{sp} - 1)}{(k_{sp} - 1)^2} \quad (3)$$

According to [38], for sandstones with relatively high porosity (between 20% and 35%), $k_{sp} \approx 0.25$ and $k_{mk} \approx 0.35$.

Fig. 3a shows an example of the plot of function (1) with the coefficients $A_i$ from Eqs. (3) and (4) defined for $P^* = 200$ MPa and coefficients $B_i$ given in the caption of this figure (the definition of these coefficients will be explained later in this section). It is seen that at $\sigma_m > \sigma^m$, $F$ grows with $\gamma^p$, meaning that the hardening modulus defined as $H = \partial F/\partial \gamma^p$ is positive (strain hardening regime). At $\sigma_m < \sigma^m$, $H < 0$ (softening) (Fig. 3b). This softening/hardening dependence on $\sigma_m$ is typical for geomaterials. The transition between these deformation regimes corresponds to the brittle–ductile transition which occurs at $\sigma_m = \sigma^m$ when $H = 0$. From the equation $H = \partial F/\partial \gamma^p = (\sigma_m^2/2)P^* - \sigma_m/B_2 - B_1$, we obtain

$$B_3 = k_{bd}k_{mk}P^*(k_{bd}k_{mk}P^*B_2 - 1)/B_2, \quad (5)$$

where $k_{bd} = \sigma^m/\sigma^m_{mk}$. According to the experimental data, $k_{bd}$ lies between $\sim 0.5$ and 1 (e.g., [30]). In this work we assume $k_{bd} = 0.7$. Thus there are still two parameters to be adjusted, the dimensionless parameter $B_2$, and $B_1$ which is proportional to $1/P^*$. They control the evolution of the yield surface with $\gamma^p$.

The crest of the yield surface in Fig. 3a is shifting toward the higher $\sigma_m$ values with inelastic strain increase. The rate of this shift $m$ defined as $m = \sqrt{3}(\partial F/\partial \sigma_m^m)$ corresponds to the slope of the “critical state line” supposed to be linear [34]. The average value of $m$ was estimated for different sandstones to vary from $\sim 0.3$ to 0.6 for the inelastic volume strain $\epsilon^p$ between 0 and $\sim 0.05$ [38]. In the simulations presented below an intermediate value of $m = 0.5$ is adopted. For the yield function (1, 2), we compute $m$ as

$$m = \sqrt{3}\frac{\tau^p(\epsilon^p = 0.05) - \tau^p(\epsilon^p = 0)}{\sigma^p(\epsilon^p = 0.05) - \sigma^p(\epsilon^p = 0)}. \quad (6)$$

Another important characteristic of the material properties is the rate of the lateral expansion (along $\sigma_m$-axis) of the yield surface with inelastic deformation and hence of the increase of $H$ with $\sigma_m$, which in any case is positive at the cap. We characterize this rate by the increase of $\sigma^m_{mr}$, the mean stress at $\tau = 0$, with deformation using the ratio

$$k_{pp} = \frac{\sigma^m(\epsilon^p = 0.05)}{\sigma^m(\epsilon^p = 0)} \quad (7)$$

which varies for different (four) sandstones from $\sim 1.15$ to 2.1 [7]. For the numerical simulations we chose an intermediate (reference) value of $k_{pp} = 1.6$, but will also present the result for the maximal value $k_{pp} = 2.1$ characterizing Adamswiller sandstone [7]. Fixing $m$ and $k_{pp}$ values allows to define the coefficients $B_1$ and $B_2$ from (6) and (7), but for this, the relation between $\gamma^p$ and $\epsilon^p$ should be established first using the plastic potential function.

3.2. Plastic potential

This can be deduced theoretically, and it will be seen in the numerical models presented hereafter that the hydrostat slope at $P_c > P^*$ is strongly dependent on the inelastic volume change defined by the dilatancy factor $\beta$ entering the plastic potential
We adopt the Drucker–Prager function,
\[ G = \tau - \beta \sigma_m \]
where \( \beta \) increases with \( \tau^p \) from the initial negative value \( \beta_0 \) approaching zero in accordance with the experimental data \[33\].
Linear function \( \beta(\tau^p) \) is applied
\[ \beta = \begin{cases} \beta_l + \beta_0(1 - \tau^p/\tau_0^p) & \text{if } \tau^p \leq \tau_0^p \\ \beta_l & \text{if } \tau^p > \tau_0^p \end{cases} \]
and the following reasonable reference values \( \beta_0 = -1, \beta_l = -10^{-3}, \text{ and } \tau_0^p = 0.1 \) are adopted, although other values were tested in the numerical models as well. The \( \beta \) value reaches almost zero \( (\beta_l) \) at \( \tau^p = \tau_0^p \), corresponding to the end of the material compaction (end of the reduction of the porosity or of the inelastic volume strain \( \varepsilon^p \)). This strain \( (\varepsilon^p) \) can be used as a parameter tracking the history of inelastic deformation in \( (1) \) instead of \( \tau^p \). The yield function \( F(\tau, \sigma_m, \varepsilon^p) \) will have a different form in this case, but the constitutive model will not

Fig. 3. Plots of the function \( F(\tau, \sigma_m, \tau^p) = 0 \) from \( (1) \) and \( (2) \) (a) and of its partial derivatives \( h = (\partial \tau / \partial \tau^p) / G \) (b) and \( \alpha = -\partial F / \partial \sigma_m \) (c) corresponding to the normalized hardening modulus \( h \) and internal friction coefficient \( \alpha \) (\( G = 5.25 \) GPa is the shear modulus). The following parameter values used in presented numerical models were adopted: \( A_1 = -2.96 \times 10^{-4} \) MPa \(^{-1} \); \( A_2 = 0.414 \); \( A_3 = 35.5 \) MPa (these are computed from \( (3) \) and \( (4) \) for \( P_n = 200 \) MPa); \( \beta_1 = 1/P_n = 1.67 \times 10^{-4} \) MPa \(^{-1} \); \( B_1 = 1; B_2 = -53 \) MPa (see text for the definition of these parameter values).
change as $\varepsilon^p$ and $\bar{\tau}^p$ are related via $\beta$

$$\varepsilon^p = - \int \beta(\bar{\tau}^p) d\bar{\tau}^p = \begin{cases} \frac{\beta_1\bar{\tau}^p - \beta_0}{\beta_0} - \beta_1 \varepsilon^p & \text{if } \varepsilon^p \leq \varepsilon_0^p \\ -\beta_1 \varepsilon^p & \text{if } \varepsilon^p > \varepsilon_0^p \end{cases}$$

or

$$\bar{\tau}^p = \begin{cases} \frac{\varepsilon^p - \varepsilon_0^p - \beta_1 - \beta}{\beta_1} \left[ 1 + \frac{\beta_0}{\varepsilon_0^p} \right] & \text{if } \varepsilon^p \leq \varepsilon_0^p \\ -\beta \frac{\varepsilon^p}{\beta_1} & \text{if } \varepsilon^p > \varepsilon_0^p \end{cases}$$

(10) into Eq. (1)) will result in a more rapid (accelerating) increase of $\tau$ with $\varepsilon^p$ than with $\bar{\tau}^p$. The physical sense of the parameter $\varepsilon_0^p$ follows thus from the fact that at this strain the porosity reduction ceases (the capacity of the material compaction is exhausted).

Having function $E(\tau, \sigma_m, \varepsilon^p)$, we can calculate $m$ and $k_{pp}$ from Eqs. (6) and (7) or inversely, define the coefficients $B_1$ and $B_2$ from the imposed values of $m$ and $k_{pp}$ that are respectively 0.5 and 1.6 as indicated in the previous section. The corresponding values of $B_1$ and $B_2$ are given in the caption of Fig. 3. Coefficient $B_1$ is calculated then from Eq. (5).

The introduced constitutive model (1)–(10) captures all principal properties of the porous rocks and more specifically of sandstones. As for rocks, the friction coefficient $\alpha$ defined as $\alpha = -\partial F/\partial \sigma_m$ decreases for this model with $\sigma_m$ and increases (first more rapidly and then slowly) with $\bar{\tau}^p$ remaining within a reasonable range, $-0.8 < \alpha < 0.8$ (Fig. 3c).

3.3. Elastic properties

The elastic properties are described by Hooke’s law. For the Poisson ratio $\nu$ we assume a typical value for sandstones of 0.2. The bulk modulus $K$ is defined by the hydrostat’s slope which is not constant at the initial loading stages corresponding to the nonlinear elasticity as indicated in Section 1. The nonlinear elastic strains are not negligible, but they are small compared to the $\varepsilon$ values at $P \geq P_m$ and do not affect the inelastic deformation (at sufficiently large $P_m$) which is of primary interest in this paper. Therefore in most of presented models we assume that $K$ is constant and corresponds to the linear segments of the hydrostats in Fig. 2. Since we are not aiming to model any particular sandstone but rather to investigate a generic case representative of sandstones, we chose the intermediate slope of the curves in Fig. 2, $K = 7$ GPa (but other values were tested as well). The corresponding values of the Young and shear moduli are $E = 12.6$ GPa and $G = 5.25$ GPa, respectively. We present also one model where $K$ increases with $\varepsilon$ from a relatively small initial value to the maximum (final) value corresponding to the linear hydrostat segment.

The formulated constitutive model has been implemented into the finite-difference 3-D dynamic time-matching explicit code FLAC3D used for the numerical modeling in this work. The simulations were carried out in large-strain mode.

4. Modeling setup

The model (Fig. 1) includes the stiff elastic platens (with the elastic moduli given in the caption of Fig. 1 and corresponding to steel) and the cylindrical specimen simulating porous sandstones with elastic–plastic properties corresponding to the constitutive model described in the previous section. The platens and specimen are separated by the frictional interfaces with zero cohesion and the interface friction angle $\phi_{int}$ varying from 0° to 10° in different models. The sizes of the specimen and platens are given in Fig. 1 caption.

The whole model (including specimen and platens) is subjected to the external normal stress $P_n$, simulating the confining pressure in the experimental tests. The pressure is quasi-statically increased from zero to a given value. During this process, the average volume strain $\varepsilon$ is calculated as the ratio of the whole specimen volume

![Fig. 4. Plot $\bar{\tau}^p(\varepsilon^p)$ from (10).](image1)

![Fig. 5. Numerical hydrostats for different model resolution $N_k$ at $\phi_{int}=10°$ (a), and for different $\phi_{int}$ at $N_k=40$ (b).](image2)
change to the initial specimen volume. The “numerical” hydrostats $P_r(\varepsilon)$ are thus defined in the same way as in the experimental tests.

5. Modeling results

All the numerical models presented below (except in Figs. 11 (curve 1) and 12) were run for the same $A_t$ and $B_t$ values given in the caption of Fig. 3. The only parameters that vary are $\beta_0$, $\gamma_0$ from (9), the interface friction angle $\phi_{int}$ and the numerical resolution $N_k$ (defined in the caption of Fig. 5). The values of these parameters are specified in the captions presenting the corresponding modeling results and in most cases are equal to the ‘reference’ values $N_k=40; \beta_0=−1, \phi_{int}=10^\circ$, and $\gamma_0=0.1$. The Poisson ratio in all models is $\nu=0.2$, and the Young modulus is $E=12.6$ GPa in all models except that in Fig. 6 (curve 1), where $E$ is twice as great.

The first models were devoted to the investigation of the impact of $N_k$ on the hydrostats. Surprisingly, this impact has proved to be practically inexistent (Fig. 5a). The same is true of $\phi_{int}$ (Fig. 5b), which was also unexpected because this parameter strongly affects the stress and strain fields as will be seen later. On the other hand, the hydrostat geometry strongly depends on the elastic parameters, the dilatancy factor and its evolution with $\eta$ (Figs. 6 and 7). Indeed, the linear segments of the hydrostats at $P_r<P_r^*$ almost coincide with line $P_r=K(\varepsilon)$ in Fig. 5a, meaning that their slopes are almost completely defined by the elastic properties (but not exactly, implying that the inelastic deformation starts well before $P_r^*$, which will be confirmed later). In Fig. 6, the curve (1) corresponds to about two times higher $E$ (and hence $K$) than for the ‘reference’ curve 2. Therefore the linear segment of the curve 1 is steeper. Comparison of curves (1) and (2) in Fig. 6 shows that $E$ also affects the hydrostat geometry at $P_r>P_r^*$. These curves become steeper again at very large $P_r$ as in real cases in Fig. 2. The steepening of the hydrostats can be shifted to larger $\varepsilon$ by increasing $\gamma_0$ (Fig. 7a). It is thus due to the reduction of $|\beta|$ with $\gamma$. Fig. 7b shows the influence of $\beta_0$ on the hydrostats geometry for constant $\gamma_0$.

An example of the evolution of $\sigma_m$, $\tau$, and $P$ fields in the specimen with $P_r$ is presented in Fig. 8. It shows very heterogeneous stress and strain fields at all the loading stages. The inelastic deformation starts very early in the loading history, at around $P_r=120$ MPa (well before reaching $P_r=P_e=200$ MPa) at the centers of the specimen ends (Fig. 8a). The contrast of the $\tau$ values in the central part and the ends of the specimen reaches almost three orders of magnitude at $P_r=150$ MPa and then reduces with $P_r$ due to the inelastic deformation (for the isotropic stress state $\tau=0$). The same is true for $P$. The heterogeneity of $\sigma_m$ is much smaller and also reduces with $P_r$ due to the inelastic deformation.

The difference of the above parameters and their evolution at different points within the specimen is presented in Fig. 9 in relation with the yield surface and for different $\phi_{int}$. It is seen that different points follow different loading paths that do not copy the path of the external loading of the specimen which coincides with the hydrostatic axis. In all the cases the stress state in all the points first evolves in the elastic domain (before touching/reaching the yields surface at different $P_r$ at different points), and then follows this surface in the elastic–plastic domain. When certain points have already been involved in inelastic straining, others can still be in the elastic domain, but they “feel” the changes in the neighboring points. The slope of the uppermost (blue) curve (curve 1) in Fig. 9b in the elastic domain, for example, is abruptly reduced around $\sigma_m=100$ MPa because of the onset of inelastic deformation at the specimen ends along its axis at the same moment (the same $P_r$ which is not equal to $\sigma_m$; $\sigma_m$ is smaller than $P_r$ at the specimen ends (Fig. 8)). The difference between the stress paths at different points decreases with $\phi_{int}$ reduction (Fig. 9). The paths practically coincide at $\phi_{int}=0$, but even in this case, $\tau$ does not completely vanish and increases with $P_r$.

Fig. 10 presents the slip histories (histories of shear displacement between the specimen and the platen) along the upper interface for different $\phi_{int}$. It shows that the slip is blocked at certain $P_r$ values, different for different points. The blockage occurs during the plastic yielding first in the axial zone of the specimen.
end and then propagates radially toward the specimen perimeter. As expected, the lower the $\phi_{\text{int}}$ value, the later the slip vanishes. The slip values, however, remain very small, of the order of 100 $\mu$m.

Fig. 11 (curve 1) shows the numerical hydrostat for the $m$ and $k_{PP}$ values corresponding to Adamswiller sandstone. As indicated above, $k_{PP}$ for this rock has a large value of $\sim 2.1$ and $m \approx 0.5$ [38]. The corresponding coefficients $B_1$ and $B_2$ defined from (6) and (7)
are $1/(0.9P_n)$ and $1/18$ respectively. Coefficient $B_3$ calculated from (5) is $-0.08P_n$. $A_i$ values are the same as in all the previous models. It is seen from Fig. 11 that the numerical hydrostat for the new $B_i$ values (curve 1) is different from the curve 2 corresponding to the previous $B_i$ values, and is closer to the experimental hydrostat for Adamswiller sandstone (curve 3).

In Fig. 12 we show one more hydrostat calculated for the Adamswiller sandstone’s parameters, where $K$ increases with $P_c$ (or with $\varepsilon$, which is the same as $\varepsilon$ is uniquely defined by $P_c$ in the hydrostatic tests)

$$K = \begin{cases} \frac{-0.48P_c^2 + 99P_c + 1428}{6500} \text{MPa} & \text{if } P_c \leq 100 \text{ MPa} \\ 6500 \text{ MPa} & \text{if } P_c > 100 \text{ MPa} \end{cases}$$ (11)

The $K/G$ ratio is constant in this model as $\nu = 0.2$ is constant. Therefore $G$ also evolves with $P_c$ (Fig. 12a). It is seen that the fit between the numerical and experimental curves became much better (Fig. 12b). It can be further improved by adjusting $P^*$ and $\beta$, but this is not our aim.

6. Concluding discussion

The most striking result of the presented models is a strong heterogeneity of the stress and strain fields within the presumably hydrostatically (isotropically) loaded cylindrical specimens (Fig. 8). These fields are characterized by strong gradients in the axial direction at the specimen ends (Fig. 8). The mean stress reduces, while the shear stress and inelastic strain rapidly increase toward the ends. These gradients grow dramatically with $\phi_{int}$ as can be deduced from the loading paths at different points within the model for different $\phi_{int}$ in Fig. 9. The gradients are nonzero even at $\phi_{int} = 0$, implying that the deviation of the stress field in the specimen from the isotropic state is caused not only by the friction at the specimen ends, but also depends on the properties (e.g., stiffness) of the bodies (platens, in our case) in contact (elastic misfit), and probably on the specimen geometry as well. This suggests that the hydrostatic state within a solid is rather exotic and can be only transitional whatever the loading conditions.

The along-axis stress gradient increases with $P_c$ in the elastic regime and reduces with inelastic strain $\gamma$ that occurs at higher $P_c$. This strain starts at the specimen ends and then involves the whole specimen (Fig. 8) so that the rate or increments of $\Delta \gamma$, $\Delta \gamma$, are first maximal at the specimen ends and then in its central part, which is seen in Fig. 13. Similar temporal and spatial distribution is typical for the acoustic emissions in the real hydrostatic tests of sandstones (Fig. 14). This is logical, since the rate (number per unit time) of the emissions is proportional to the rate of material damage, which in the model is represented by the inelastic strain. Like $\Delta \gamma$ in the numerical models, the rate of the acoustic emissions is maximal at the specimen ends only until certain, close to $P^*$, $P_c$ values, after which one observes a lack of the material damage rate at the specimen ends and its increase within the central part (Fig. 14). It seems also that the emissions within the loading segments (c) and (d) are concentrated along the...
peripheral part of the specimen (compared to its axial part), which is in agreement with the distribution of $\Delta \gamma_p$ in Fig. 13 (segments (c) and (d)). Within the segments (a) and (b) in Fig. 14 one observes zones of localization of the acoustic emissions. They are certainly caused by the preexisting heterogeneities in the sandstone specimen. The presented numerical models are completely homogeneous.

The acoustic emissions thus confirm the modeling results regarding the heterogeneity of the strain (and hence stress), including its along-axis gradient changing with loading in the real hydrostatic tests in the same way as in the models. All this suggests that the external stress and strain measurements bring only approximate information about the actual stresses and strains in a specimen. In spite of this, the global external measurements allow a quite accurate determination of $P^*$ (e.g., Figs. 5 and 6), which is very close to the $\sigma_m$ value at the intersection of the initial yield envelope with the hydrostatic axis equal to 200 MPa (Fig. 3a).

Global measurements at $P_c < P^*$ also provide an accurate definition of the $K$ value, which follows from Fig. 5a. The hydrostat geometry at $P_c > P^*$ also depends on $K$ (Fig. 6), which is supposed to be constant, but is mainly defined by $\beta$ and its evolution with $\gamma_p$ (Fig. 7). The steepening of real hydrostats at very high $P_c$ (for a given material), is due to the increase of $\beta$ with $\gamma_p$ and its approaching zero, which follows from the models in Fig. 7. The $\beta(\gamma_p)$ dependence can be constrained therefore by adjusting the numerical hydrostats to the measured ones, but the curve fitting is not the aim of this work. Note that the slopes of the steep segments of real hydrostats at very high $\sigma_m$ are systematically lower than the linear-hydrostat-segment slopes at $\sigma_m = P^*$ (Fig. 2).

The results presented do not suggest any need for a double yield surface composed of what is sometimes called the Coulomb and cap segments intersecting near the $\tau$ peak (with vertex or without) and characterized by different hardening laws.
Fig. 12. Evolution of the elastic moduli with $P_c$ according to Eq. (11) (a) and the corresponding numerical hydrostat (b, curve 1) from the model run under the same other conditions as the model corresponding to Adamswiller sandstone in Fig. 11 (curve 1 in Fig. 11 and curve 2 in Fig. 12). Also shown is the experimental hydrostat for this sandstone (curve 3).

Fig. 13. The incremental inelastic strains $\Delta \gamma^p$ accumulated during the loading intervals (segments) indicated on the numerical hydrostat. $N_k = 60$, $\phi_{int} = 10^\circ$, $\beta_k = -1$, and $\gamma_0^p = 0.1$.

Fig. 14. Distribution of the acoustic emissions generated during loading segments indicated on the hydrostat for Bleurswiller sandstone [39].
The next step in the presented research is the modeling of the deviatoric triaxial tests. The obtained results suggest that the end-effects will also impact the deformation under deviatoric loading. The concentration of shear stress $\tau$ at relatively small $P_c$ and of the inelastic strain (compaction) at large $P_c$ (or $\sigma_{nn}$) at specimen's ends will produce different effects at different $P_c$ (high $\tau$ will promote the initiation of the specimen failure at its ends, while strong compaction (hardening) at the ends will inhibit such initiation).

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References